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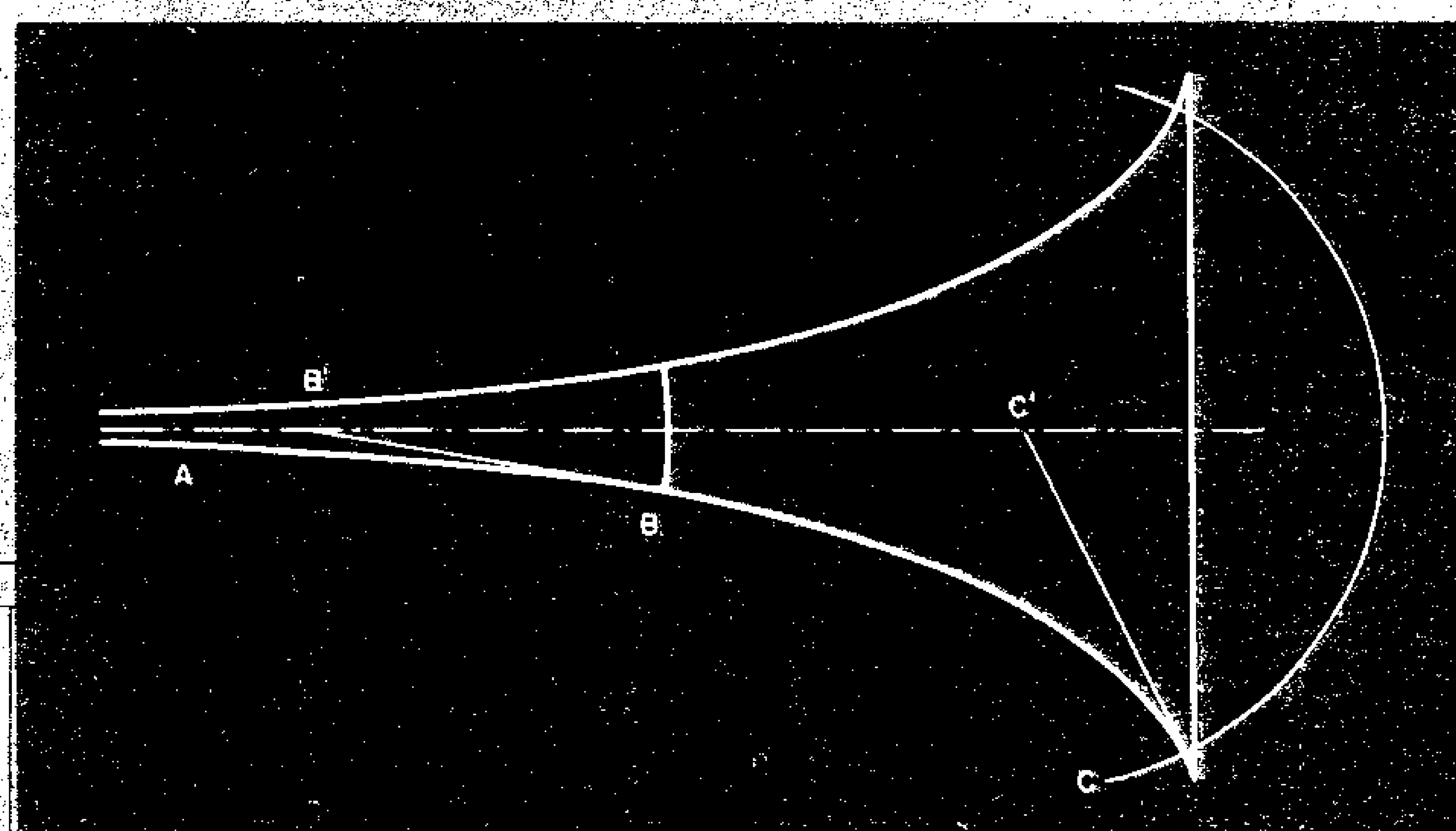

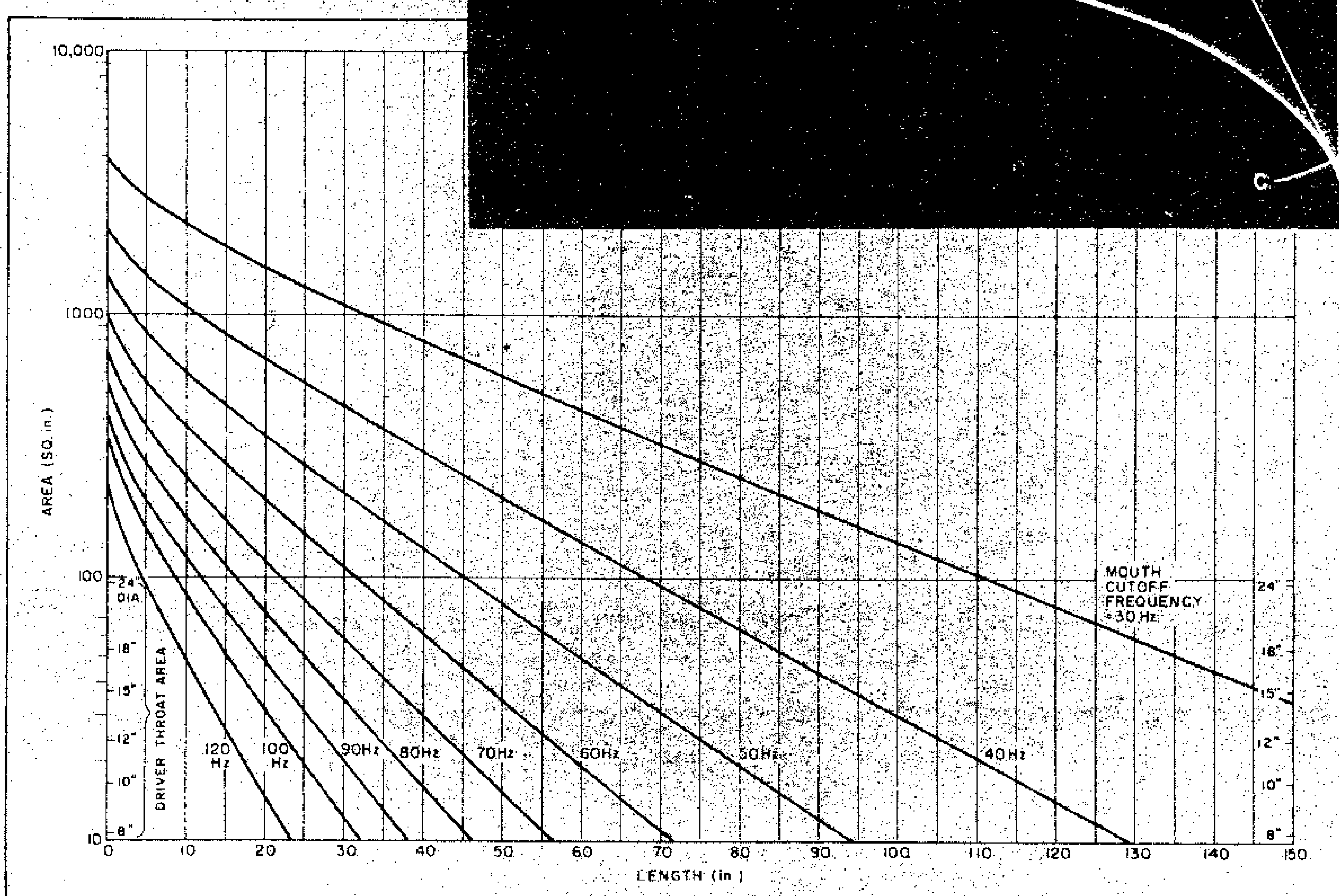

The Tractrix Horn Contour

by Bruce C. Edgar

Introduction

In his 1974 article for *Wireless World* on horn loudspeakers, Dinsdale[1] introduced the present generation of speaker builders to the tractrix horn contour. The tractrix curve, he claimed, combined the excellent low frequency characteristics of the exponential curve with the spherical wave propagation characteristics of the conical horn.

"Well," you may ask, "if the tractrix contour was so great, why has it been ignored for the last 40 years?"

The principal reason for the tractrix contour's relative obscurity is probably the complexity of its mathematical expression. A non-mathematician would have great difficulty using it, and even those skilled with programmable calculators may shy away from it.

Before the advent of the digital computer, engineers did much of their nuts and bolts design work with a simple aid: the design curve or chart. Back in 1938, Sanial[2] assembled a series of design curves for the exponential horn. In the same manner, we will evolve some tractrix horn design contours so you can design your own tractrix horns without too much difficulty.

Some History and Theory

Webster[3] assumed in his 1919 pioneer paper on horns that the wavefronts in an exponential horn are plane (no curvature). Hanna and Slepian[4] later realized that the plane wave assumption was not valid at low frequencies. In 1934, Wilson[5] proposed a modified exponential horn in which the wavefronts made a gradual transition from plane to spherical waves.

Independent of all the theoretical analysis on how horns work, a 24 year old British inventor, P.G.A.H. Voigt started in 1926 to design a moving coil loudspeaker. He argued that a loudspeaker should be as efficient as possible and that this goal could only be approached by using the maximum practical field strength in the magnet. Using an 80 lb. iron electromagnet driving a 6" diaphragm, he found that the test results were very disappointing. The sound was very "tinny." He reasoned that since the diaphragm radius at low frequencies was a small fraction of a wavelength, the air, instead of resisting the diaphragm, was "escaping" sideways and did not load the speaker.

Voigt saw clearly that attaching something as simple as a straight pipe to the diaphragm would not do. When a wavefront sees a discontinuity at the end of the pipe, a reflection of the wave occurs which travels back to the diaphragm and tends to make the pipe resonate much like an organ pipe. Voigt reasoned that the pipe should be expanded very slightly near the diaphragm, and as the wavefront moved away from the diaphragm the tapering angle could be increased gradually. He also recognized that the wavefront at the wall will try to follow it and simultaneously be slowed due to friction. These two processes naturally produce a rounded wavefront since its center would be least affected by the wall.

If the pipe's tapering angle is increased until the taper is at 90° in relation to the axis, the wavefront becomes a hemisphere which matches nicely the outside air's tendency for spherical expansion of wavefronts from a source.

Voigt, in a letter to the author (7 Jan. 1981), says:

> "As I drew out this curve to make the smoothest possible transition from the nearly parallel taper near the diaphragm to a 90° angle to the axis, I wondered if I had re-invented the standard exponential curve mentioned in some advertisements (this being the mid-1920's). When I plotted the latter I found that at the throat where the taper was very slight, the difference was negligible. As the mouth was approached, however, the taper increased faster than the exponential, and the 90° angle was reached quite soon so that it seemed shorter (see *Fig. 1*). Later I learned from our draftsman that the curve was known in the mechanical world and that its name was a Tractrix."

In July 1926 Voigt applied for a patent on the tractrix horn contour and British patent #278,098 was granted to him in 1927. In the early 1930's Voigt[6] introduced a commercial horn utilizing the tractrix contour and an electrodynamic driver of his own design. The horn was 4 ft. long with a mouth opening 4 ft. square. The field coil (remember, this was before the days of good permanent magnets) required 40 watts DC, but the horn efficiency was so outstanding it required only 4 watts to fill an audiotorium. The horn's response was good down to 100 Hz with some response still audible at 50 Hz (letter from Voigt, 7 Jan. 1981). Many of these tractrix horns were used in British cinemas through World War II.

In 1934 Voigt introduced his corner horn[7,8] for domestic use which was an adaptation of his commercial tractrix horn. The speaker (see *Fig. 2*) featured a 4 ft. square mouth and a tapered

quarter wave length pipe which supplied the bass response below 100 Hz. A contemporary critic, Percy Wilson[9,10] commended the Voigt horn for its fine sound, but could not understand why the tractrix contour was better than the modified exponential contour which Wilson championed. After Voigt's pioneering work in the twenties and thirties, Jensen and Lambert[11] examined the tractrix horn in 1954 and concluded after much mathematical analysis that the design was a valid alternative to the exponential horn. However they did not mention Voigt's work. No one picked up on the research, and it remained dormant until the Dinsdale review in 1974. So the amateur speaker builder has plenty of room to experiment with the tractrix horn.

Tractrix Horn Contour

Figure 3 shows how a spherical wavefront propagates down a tractrix circular horn. The spherical wavefront has a constant radius arm (B′B = C′C) such that the radius arm is always tangent to the wall of the horn. This fact can be used to graphically generate a tractrix curve, as shown by Dinsdale[1], but I found this procedure too error-prone, although doing one curve by hand is fun. For serious design work it is best to work from the tractrix mathematical expression as derived in the appendix.

The tractrix curve is given by:

$$x = a \cdot \ln\left(\frac{a+\sqrt{a^2-r^2}}{r}\right) - \sqrt{a^2-r^2}$$

where x = distance along the axis measured from the mouth,
a = radius of the mouth,
r = radius of the horn at point x

This expression is somewhat awkward to use because we cannot plug in a value for x and conpute r as we can for an exponential horn. However, near the throat (a>>r) tractrix expression reduces to

$$r = 2\,a\,e^{-(1+\frac{x}{a})}$$

which shows the exponential characteristic near the throat necessary for good low frequency transmission.

The mouth size determines the horn's low frequency cutoff. The cutoff condition is given by $\lambda = 2\pi a$, where λ = wavelength. The cutoff frequency is then:

$$f = \frac{c}{2\pi a}$$

where c = velocity of sound (13500 in/sec).

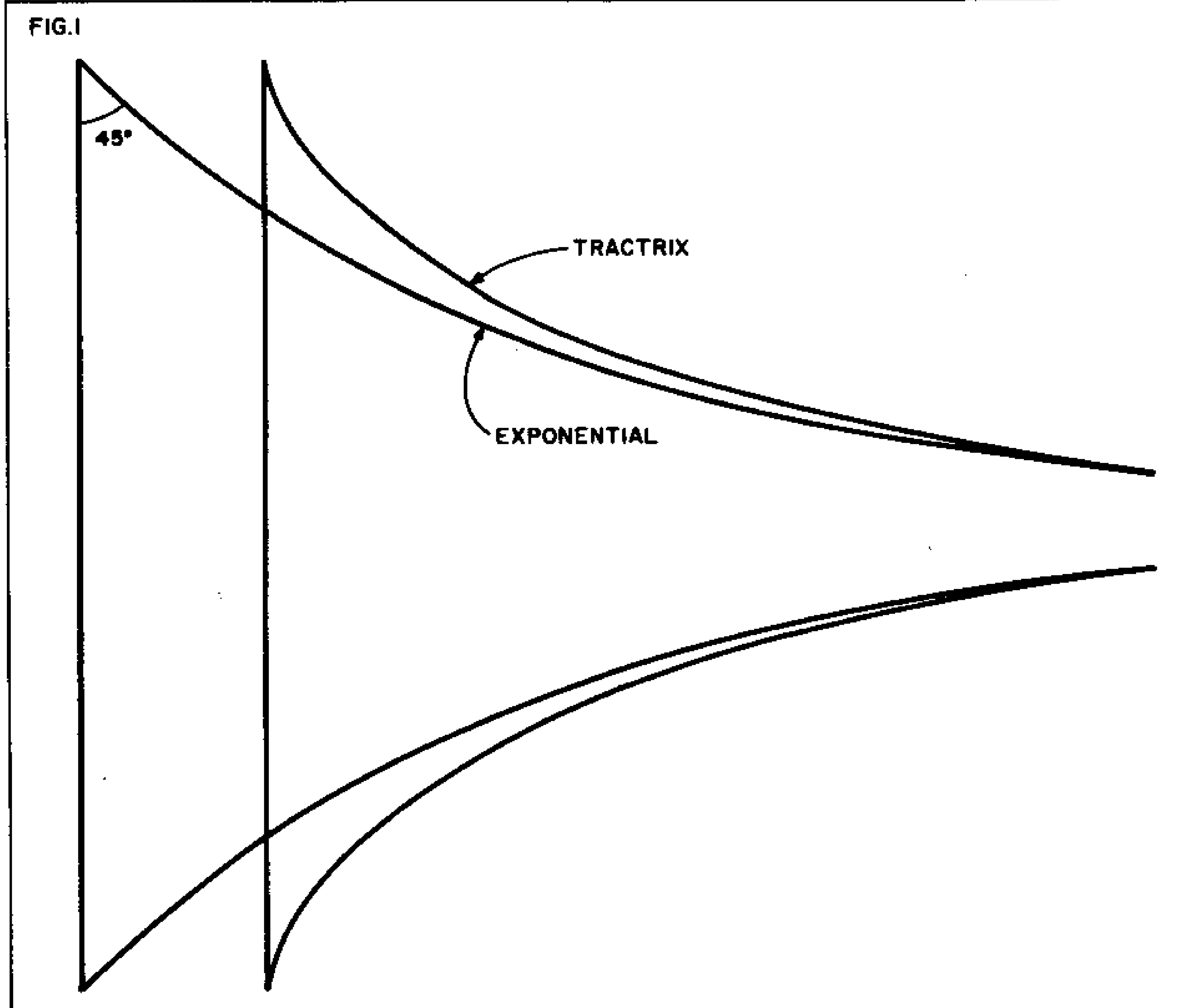

Fig. 1. Comparison of exponential and tractrix circular cross section horns normalized to the same throat and mouth areas.

Since the tractrix formula is for a circular cross-section horn and we usually construct rectangular cross-section horns, we have to convert the radius information to area by the familiar formula for the area of a circle:

$$A = \pi r^2,$$

which is good for free standing horns. For wall position horns, we divide the area by 4; for corner horns, by 8.

Some error is involved in translating the circular cross-section to rectangular because in the latter the tangent to the corner is longer than that to the sides. Voigt argues that if you make the shorter tangent in a square horn the tractrix tangent, you end up with an area of $4/\pi$ = 1.27 times that of the

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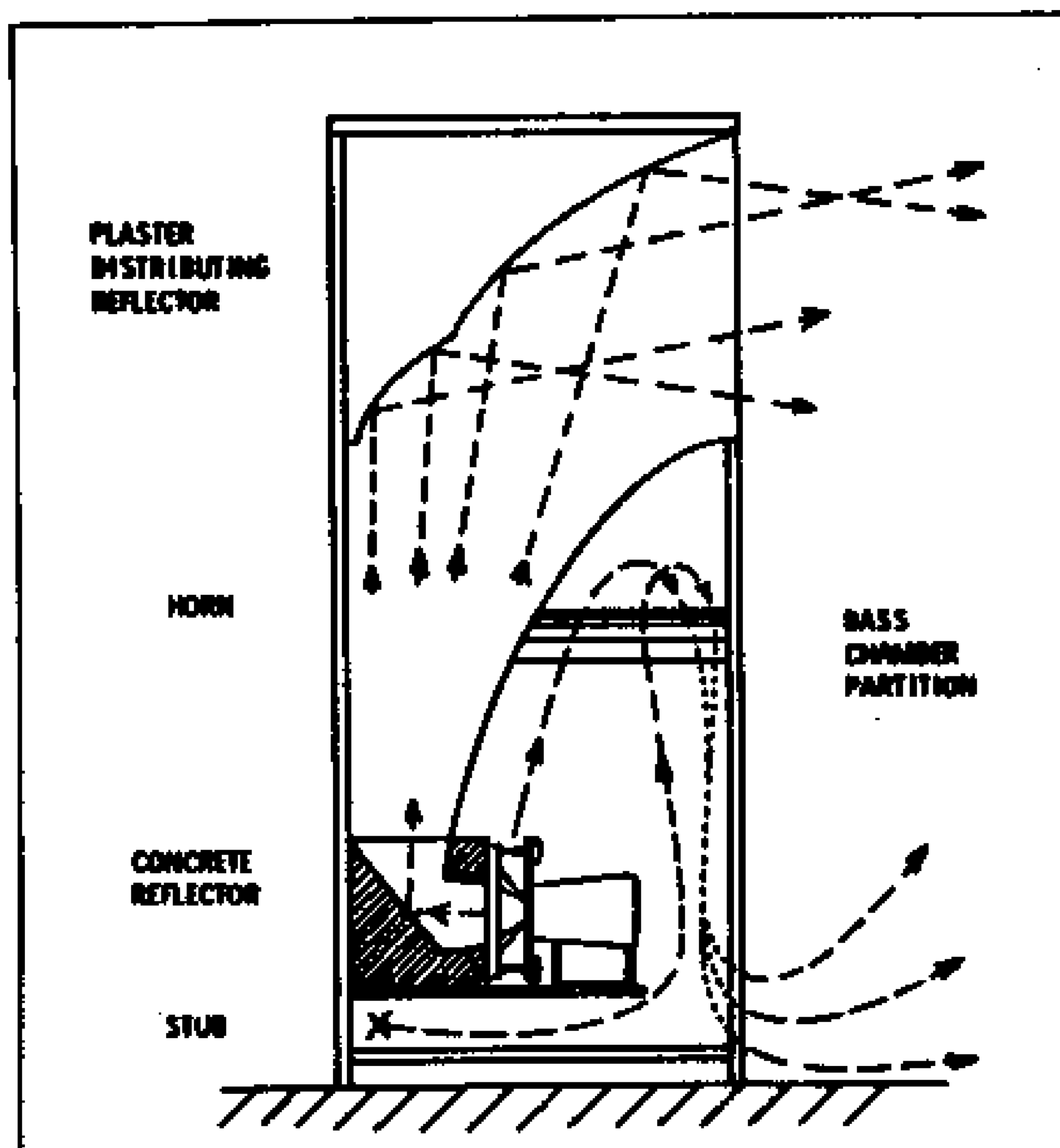

Fig. 2. Voigt corner horn, circa 1936.

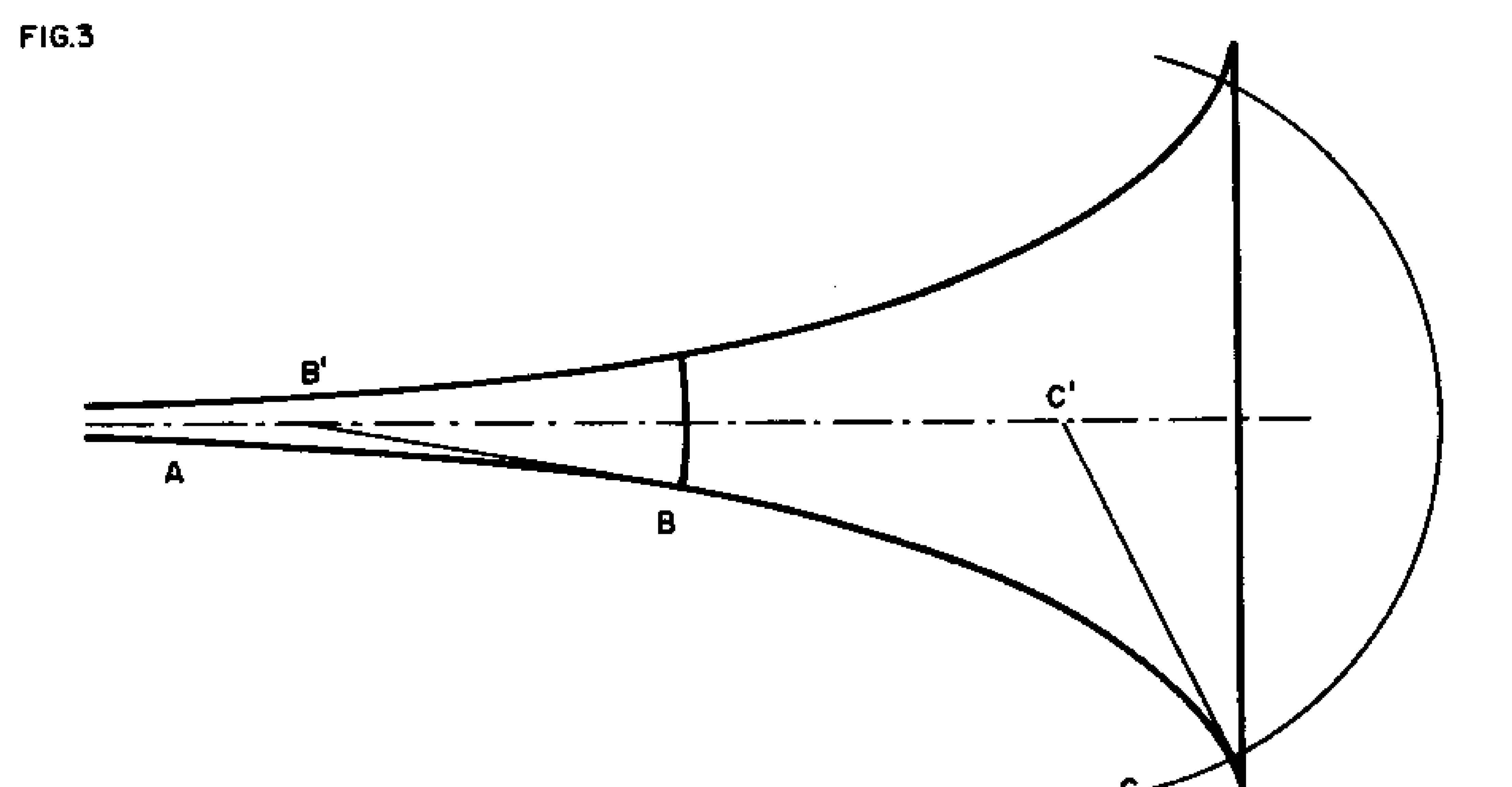

Fig. 3. Spherical wavefronts in a tractrix horn.

FIG. 4

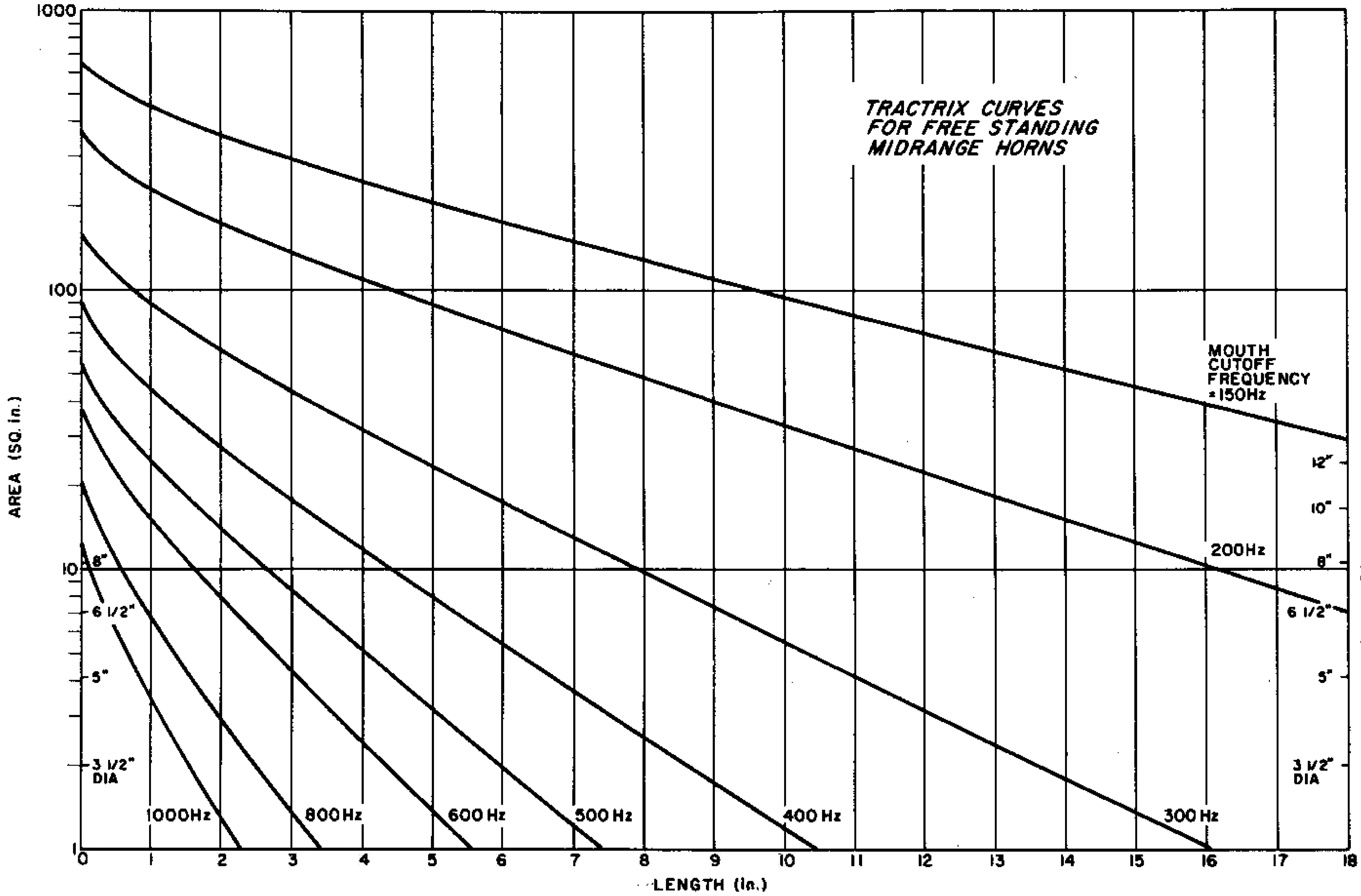

Fig. 4. Tractrix area expansion for midrange horns (free standing) with low frequency cutoffs from 150Hz to 1kHz.

FIG. 5

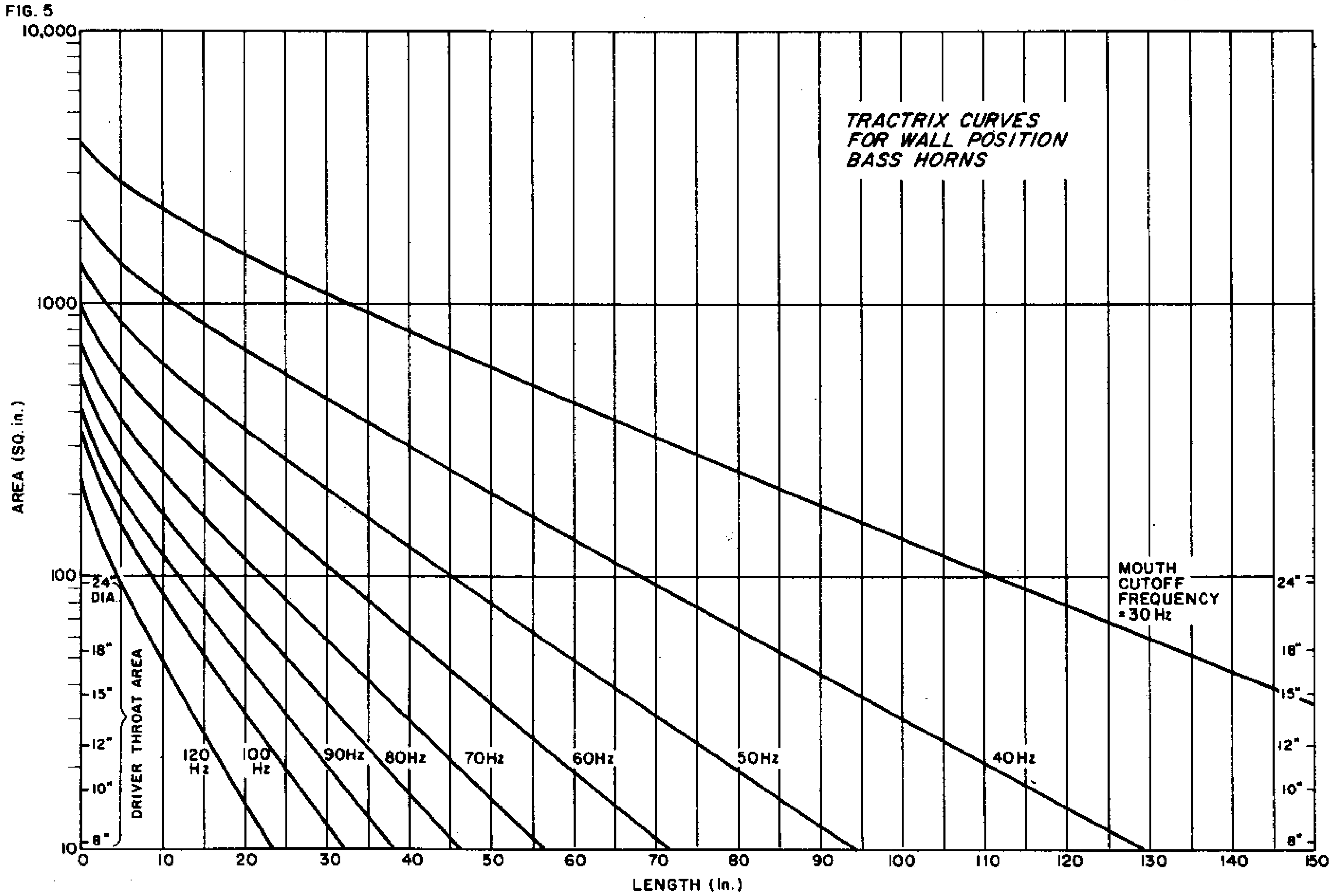

Fig. 5. Tractrix area expansion for bass horns (wall position) with low frequency cutoffs from 30Hz to 120Hz.

corresponding (circular) tractrix horn. "If the area is made equal to the corresponding tractrix, the tangent to the sides will be short, a defect which is partly compensated for by the excess length of the tangent to the corners."[12]

Design Curves

In *Figs. 4, 5,* and *6* we plot the tractrix area expansion as a function of length from the mouth for free standing horns (150Hz-1000Hz), wall position horns (30Hz-120Hz), and corner horns (30Hz-120Hz), respectively. To use these charts, simply select the desired cutoff frequency and pull out the area values every 2" to 5" (depending on the total length) down to the desired throat size. We have marked the throat areas for several speaker sizes as recommended by Dinsdale![1] However, these throat sizes were based on geometrical considerations (throat to piston area ratio of 0.33) and ignore the driver parameters which is the other half of the design problem. The reader is referred to the horn design papers of Keele,[13] Small,[14] and Leach,[15] which deal with the selection of throat size for maximum bandwidth or maximum efficiency. Generally, for maximum bandwidth of a horn, one uses throat to driver ratios of 0.50 to 0.30; for maximum efficiency

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Fig. 7. Spherical-wave midrange horn with direct radiator woofer used in Germany in the early 1950's.

FIG. 6

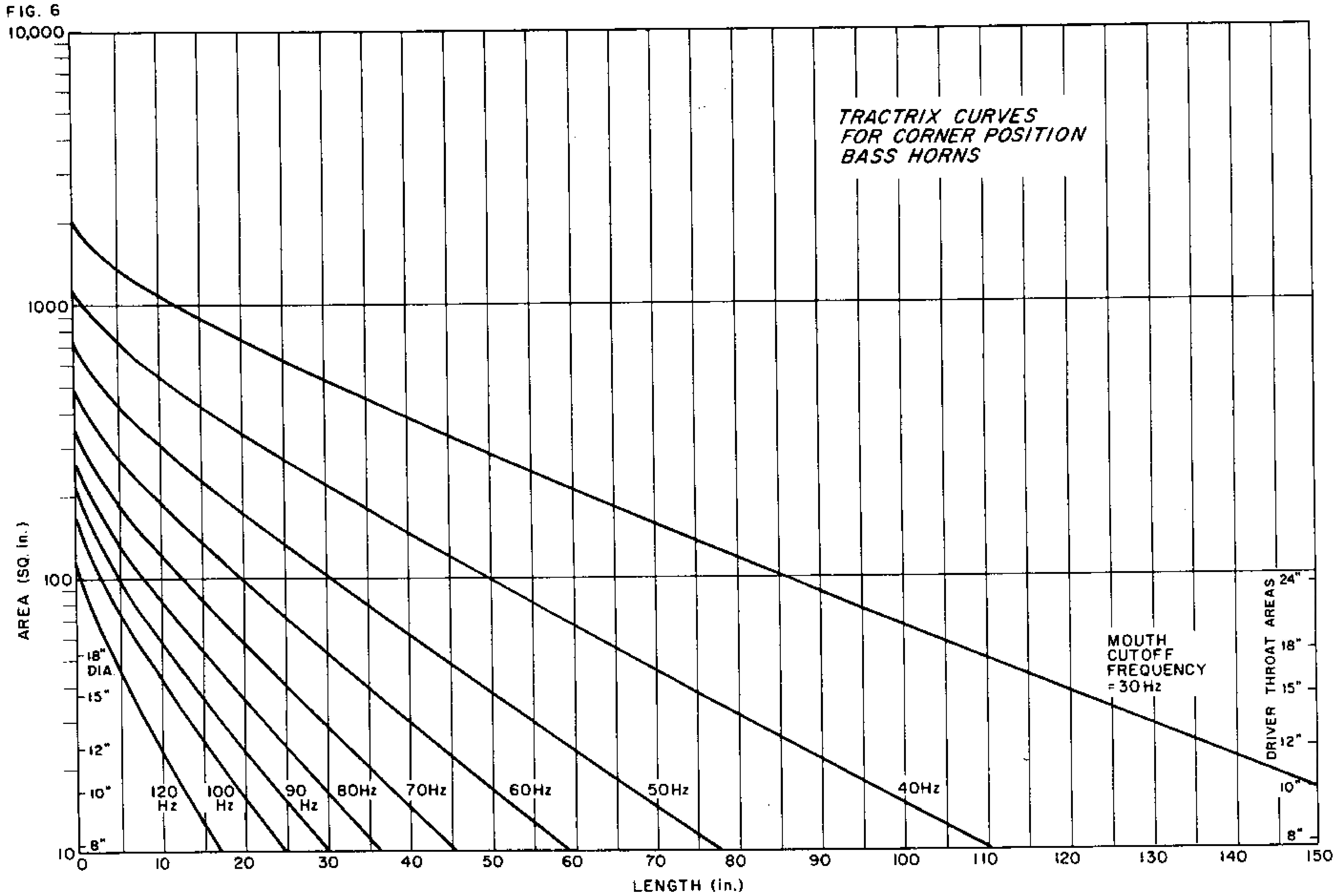

Fig. 6. Tractrix area expansion for bass horns (corner position) with low frequency cutoffs from 30Hz to 120Hz.

one uses ratios of 0.50 to 0.70.

Do not reduce the size of the mouth too much from the values of the charts. In his examples of tractrix horn design Dinsdale[1] terminates his horns just before the 90° flare is reached, resulting in a mouth area reduction of 70 percent. Lambert[11] in his analytic study of the tractrix horn indicates that the true cutoff occurs at the 80 percent point. Keele's study of optimum mouth size[16] shows that, depending on the solid radiation angle of the horn, the mouth reduction ranges between 70 and 80 percent. Probably in most cases the mouth size is determined by the folding geometry and "what fits." However, do not go below the 70-80 percent reduction factor. If you do, the resultant speaker in the case of a bass horn will become more an acoustical labyrinth and less a true horn.

Another attractive feature of tractrix horns is the 90° flare at the mouth. Keele[17] found that beaming effects found in mid-range conical horns can be minimized by doubling the flare at the mouth. The effect of additional flare is to make these conical horns look very much like a tractrix horn. The tractrix's reduced length compared to conical and exponential horns of the same frequency range gives it the advantage of being smaller than other horn contours.

Commercial Tractrix Horns

I recently ran across an example of tractrix horns (referred to as spherical-wave horns) in a book edited by Richardson.[18]

Figure 7 shows a mid-range spherical-wave horn with a direct radiator woofer; it probably has a low frequency cutoff around 300-400Hz. The interesting square cross-section construction should suggest some construction techniques to you. *Figures 8a* and *8b* show front and back views of a full range horn cluster, German-built for large cinemas in the early 1950's. Comparing the mouth size to the person standing, a rough calculation gives a lower cutoff of 25Hz for the bass horn.

Summary

I hope the tractrix design curves will stimulate constructor interest in designing and building tractrix horns. The shorter length and lack of beaming

Appendix: Tractrix Derivation

THE TRACTRIX EXPRESSION can easily be derived by anyone with a knowledge of first year calculus, but in the literature[1,9,19] you will find several different equations all purporting to be the tractrix expression.

Any point along a tractrix circular horn (*Fig. 9*) you can draw a tangent line from the horn wall to the X-axis which defines the radius of the spherical acoustic wave. From the triangle relations, the slope at (x, r) must be:

$$\frac{dr}{dx} = -\frac{r}{\sqrt{a^2-r^2}}$$

$$\text{Integrating, we find } \int \frac{\sqrt{a^2-r^2}}{r}\,dr = -\int dx$$

$$\text{or } x = a \cdot \ln\left(\frac{a+\sqrt{a^2-r^2}}{r}\right) - \sqrt{a^2-r^2}$$

We used the above expression, which is equivalent to Dinsdale's[1] tractrix expression if $a = \lambda/2\pi$ is substituted. Noting that:

$$\frac{a+\sqrt{a^2-r^2}}{a-\sqrt{a^2-r^2}} = \frac{a+\sqrt{a^2-r^2}}{a-\sqrt{a^2-r^2}} \cdot \frac{a+\sqrt{a^2-r^2}}{a+\sqrt{a^2-r^2}} = \left(\frac{a+\sqrt{a^2-r^2}}{r}\right)^2$$

we arrive at the expression used by Baldock[19] and Wilson[9]:

$$x = \frac{a}{2}\ln\left(\frac{a+\sqrt{a^2-r^2}}{a-\sqrt{a^2-r^2}}\right) - \sqrt{a^2-r^2}$$

According to Lockwood[20] the tractrix curve was generated by Leibniz in 1690, but Huygens first solved it analytically and gave the curve its name.

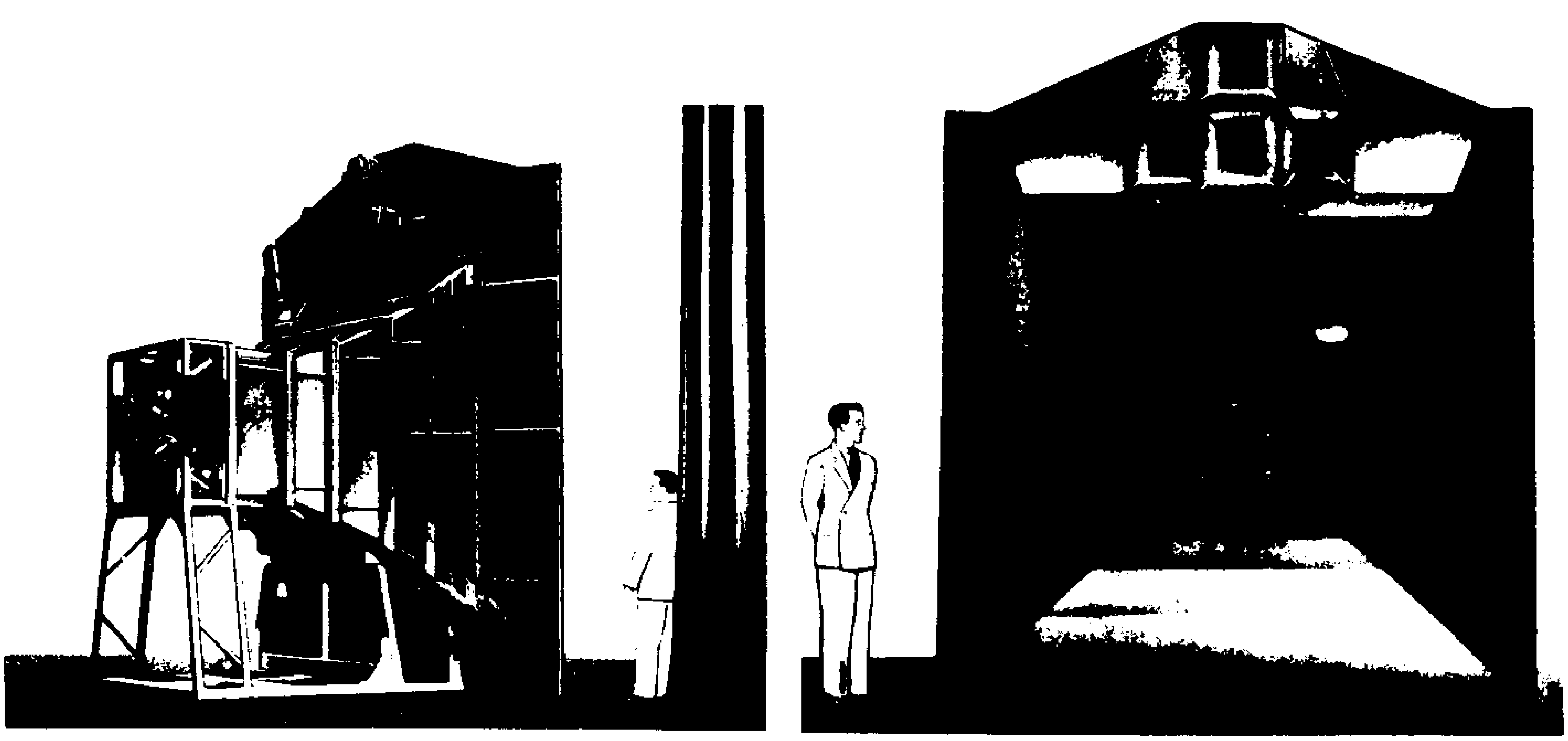

Fig. 8. Spherical-wave bass and midrange horn combination used in German cinemas in the early 1950's.

effects found in tractrix horns compared to the traditional exponential horns are two features with much to recommend them. I am currently working on several tractrix horn designs which I will write about in future issues of *Speaker Builder*. □

Acknowledgements

The author acknowledges the help of Paul Voigt in clarifying many of the points in the development of tractrix horns. In particular, he wrote me a long letter about my initial tractrix paper in January 1981, just before his untimely death, congratulating me on "the tremendous amount of library research" and at the same time upbraiding me for glossing over some historical dates. So I dedicate this article to the late Paul Voigt, a grand old gentleman which I had the pleasure of knowing during the past year. I also thank Geoffrey Wilson for many helpful comments.

The Author

Dr. Bruce C. Edgar is a space scientist for The Aerospace Corporation, El Segundo, California. His hobbies include woodworking, horn design and construction, and bicycling.

REFERENCES

1. Dinsdale, J., "Horn Loud Speaker Design," *Wireless World, 80,* 19-24 (Mar), 133-138 (May), 186-190 (June), 1974.
2. Sanial, A.J., "Graphs For Exponential Horn Design," *RCA Review, 3,* 97-104, 1938.
3. Webster, A.G., "Acoustical Impedance, and the Theory of Horns and of the Phonograph," *Proc. Nat. Acad. Sci., 5,* 275-282, 1919, reprinted in *J. Audio Eng. Soc., 25,* 24-28, 1977.
4. Hanna, C.R., and J. Slepian, "The Function and Design of Horns for Loudspeakers," Trans. A.I.E.E., 43, 393-404, 1924, reprinted in *J. Audio Eng. Soc., 25,* 573-585, 1977.
5. Wilson, P., "Technical Talk," *The Gramophone, 11,* p. 455-456 (April) p. 492-493 (May), 12, p. 30-31 (June), 1934.
6. The Voigt Loudspeaker, *The Gramophone, 10,* 289 (Dec.), 1933.
7. Wilson, P., "Technical Talk, *The Gramophone, 14,* 354, 1937.
8. Wilson, P., and G. L. Wilson, "Horn Theory and the Phonograph," *J. Audio Eng. Soc., 23,* 194-199, 1975.
9. Wilson, P., "Tractrix Horns," *The Gramophone, 12,* 119-120, 1935.
10. Wilson, P., "The Voigt Domestic Speaker," *The Gramophone, 12,* 510, 1935.
11. Lambert, R.F., "Acoustical Studies of the Tractrix Horn, I," *J. Acous. Soc. Am., 26,* 1024-1028, 1954. Jensen, A.O. and R. F. Lambert, Acoustical Studies of the Tractrix Horn, II, *J. Acous. Soc. Am., 26,* 1029-1033, 1954.
12. Voigt, P.G.A.H., British Patent #278,098, "Improvement in Horns for Acoustic Instruments," 1927.
13. Keele, D.B., "Low Frequency Horn Design using Thiele-Small Driver Parameters," *A.E.S.*, preprint #1250, 1977.
14. Small, R. H., "Suitability of Low Frequency Drivers for Horn loaded Loudspeaker Systems," *A.E.S.*, preprint #1251, 1977.
15. Leach, W.M., "On the Specification of Moving Coil Drivers for Low-frequency Horn-loaded Loudspeakers," *J. Audio Engr. Soc., 27,* 950-959, 1979.
16. Keele, D.B., "Optimum Horn Mouth Size," Audio Engineering Society preprint #933, 1973.
17. Keele, D.B., "What's So Sacred about Exponential Horns?"; Audio Engineering Society preprint #1038, 1975.
18. Richardson, E.G., ed., *Technical Aspects of Sound,* D. Van Nostrand, N.Y., 1953, p. 364-367.
19. Baldock, Rex, "Horn Acoustics," *HiFi News & Record Review, 11,* (April) 1176-1195, 1967.
20. Lockwood, E.G., *A Book of Curves,* Cambridge Univ. Press, Cambridge, 1961, p. 119-124.

FIG. 9

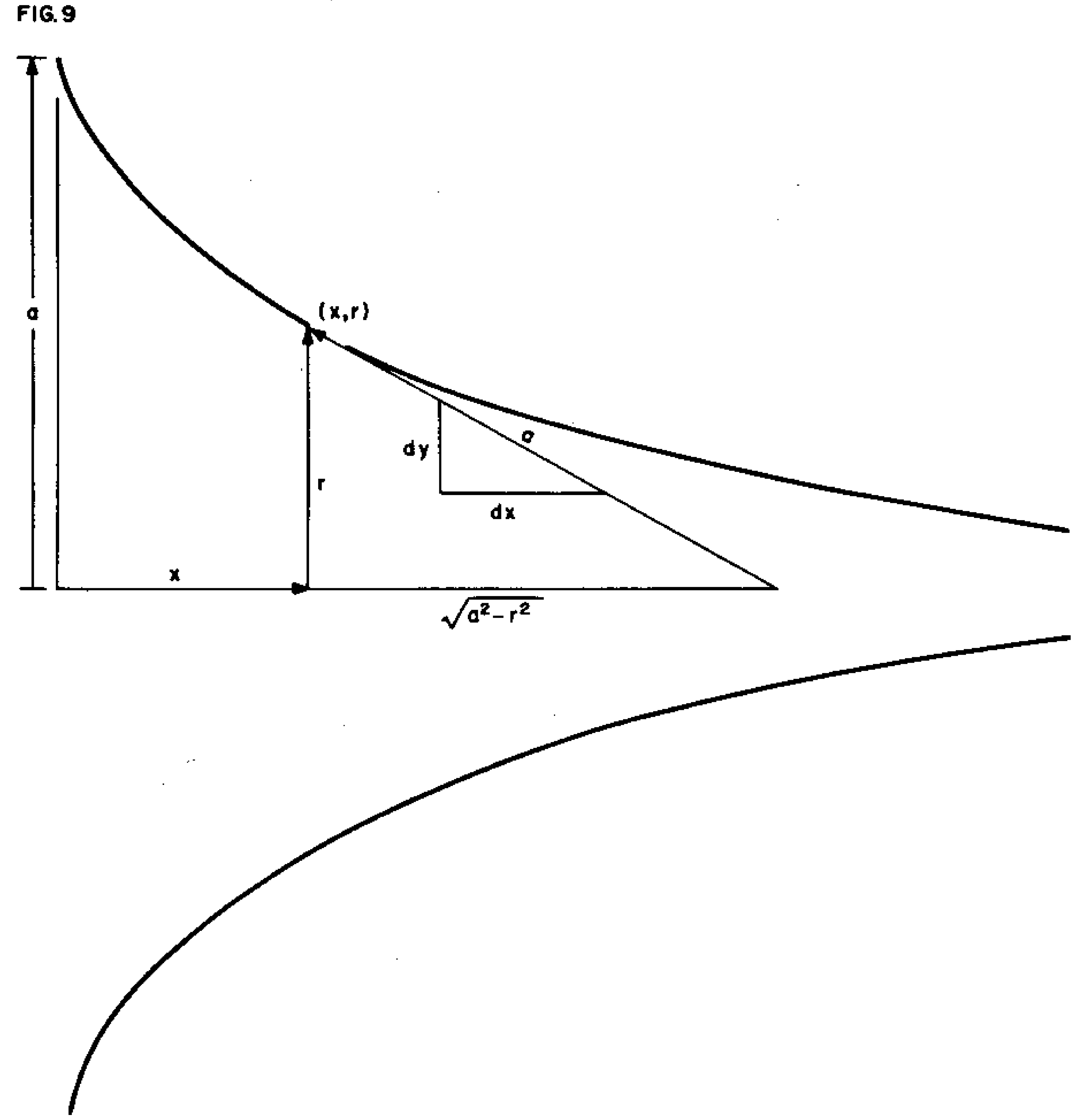

Fig. 9. Generation of tractrix contour.

Literature

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major modifications to the crossover.

McKenzie, Angus. *FM Radio.* ***Hi-Fi News*** 25:10 (Oct. 1980), p. 69. Discussion of car stereo, including placement of speakers; installation in the doors is much preferred to installation in the rear.

Preis, D. ***IEEE Transactions on Acoustics, Speech, and Signal Processing*** 28:6 (Dec. 1980), p. 769. Review of Martin Colloms, *High-performance loudspeakers* (Halsted Press, 1980, 2nd ed.). "A superb job"; "a sensible well written compendium of information for the non-specialist"; "written at a level understandable to junior or senior engineering students".

von Recklinghausen, Daniel R. *Dynamic equalization for loudspeakers.* ***Journal of the Audio Engineering Society*** 28:11 (Nov. 1980), pp. 770-774. 3 refs. The author is with KLH Research & Development Corp. A method to obtain the flattest possible low-frequency response at a given power input level, up to the large-signal limit of the driver. This system is used in KLH's Computer Controlled speakers.

Whyte, Bert. *Behind the scenes.* ***Audio*** 64:10 (Oct. 1980), pp. 6, 10, 12, 14, 16, 121. No refs. Review of new loudspeakers by IMF, B & W, Meridian, KEF, Fulton, Cerwin-Vega, ADS, and VMPS-Itone heard at the Summer Consumer Electronics Show.

Whyte, Bert, *Behind the scenes.* ***Audio*** 64:12 (Dec. 1980), pp. 112-118. Report on the design of the B & W 801 speaker.

(No author given). *Annual equipment directory.* ***Audio*** 64:10 (Oct. 1980) pp. 67-102 list commercially available loudspeakers, with price and other information. Headphones are listed pp. 118-120. Manufacturers' addresses are given pp. 158-164.

(No author given). *IEC issues two new standards on loudspeaker requirements.* ***Journal of the Acoustical Society of America*** 68:5 (Nov. 1980), p. 154.

(No author given). *Sixth annual car stereo directory.* ***Audio*** 64:7 (July 1980), pp. 28-48. Speakers are listed on pp. 35-42; specifications and prices are given. Addresses of the manufacturers are given on p. 28.